Menofia University Faculty of Engineering Shebien El-kom Basic Engineering Sci. Department. Academic Year : 2016-2017 Date : 31 / 5 / 2017	Minoufia University	Subject : Numerical Analysis Code: BES 601 Time Allowed : 3 hours Year : Master Total Marks: 100 Marks

Answer all the following questions: [100 Marks]

Q.1 **(A)** Consider the following three-dimensional Helmholtz equation in the following form: [25]

$$a\frac{\partial^2 u}{\partial x^2}+b\frac{\partial^2 u}{\partial y^2}+\lambda u=F(x,\ y,z),$$

with initial conditions:

$$u(0,y)=f_1(y),\quad u_x(0,y)=f_2(y),$$

$$u(x,0)=f_3(x),\quad u_y(x,0)=f_4(x).$$

Where;

$F(x,y)$, $f_1(y)$, $f_2(y)$, $f_3(x)$, $f_4(x)$ and a, b, λ are given functions and given constant respectively.

Solve the two-dimensional Schrodinger equation using *the differential transform method* (*DTM*), in the following form:

$$F(x,y,z)=(12x^2-3x^4)\sin(y).$$

$$a=b=1,\quad \lambda=-2, \text{ and } f_1(y)=0,\quad f_2(y)=0.$$

(B) Consider the following three-dimensional Helmholtz equation in the following form:

$$a\frac{\partial^2 u}{\partial x^2}+b\frac{\partial^2 u}{\partial y^2}+c\frac{\partial^2 u}{\partial z^2}+\lambda u=F(x,\ y,z),$$

with initial conditions:

$$u(0,y,z)=f_1(y,z),\quad u_x(0,y,z)=f_2(y,z).$$

$$u(x,0,z)=f_3(x,z),\quad u_y(x,0,z)=f_4(x,z).$$

$$u(x,y,0)=f_5(x,y),\quad u_z(x,y,0)=f_6(x,y).$$

Where;

$f_1(y,z)$, $f_2(y,z)$, $f_3(y,z)$, $f_4(y,z)$, $f_5(y,z)$, $f_6(y,z)$ and a, b, c, λ are given functions and given constant respectively.

Solve the three-dimensional *Helmholtz equation* using *the differential transform method* (*DTM*), in the following form:

$$F(x,y,z) = (12x^2 - 4x^4)\sin(y)\cos(z).$$

$$a = b = c = 1, \quad \lambda = -4, \text{ and } f_1(y,z) = 0, \quad f_2(y,z) = 0.$$

(C) Consider the nonlinear singular initial value problem:

$$y'' + \frac{2}{x}y' + 4(2e^{y} + e^{y/2}) = 0$$

with initial conditions:

$$y(0)=0, \quad y'(0)=0$$

Solve the nonlinear singular initial value problem using the adomian decomposition method (ADM).

Q.2 **(A)** Consider the following non-homogenous differential system: [25]

$$\frac{dx}{dt} = z - \cos(t), \qquad \frac{dy}{dt} = z - e^{t}, \qquad \frac{dz}{dt} = x - y$$

with initial conditions:

$$x(0) = 0, \quad y(0) = 0, \quad z(0) = 2$$

Solve the non-homogenous differential system using the differential transform method (DTM).

(B) Consider the following systems of non-linear differential equations:

$$\frac{dx}{dt} + \frac{dy}{dt} + x + y = 1, \qquad \frac{dy}{dt} = 2x + y.$$

with initial conditions:

$$x(0) = 0, \quad y(0) = 1$$

Solve the non-linear differential systems using *the differential transform method* (*DTM*).

(C) The governing equation of a uniform Bernoulli–Euler beam under pure bending resting on fluid layer under axial force is:

$$EI\frac{\partial^4 v}{\partial x^4} + p\frac{\partial^2 v}{\partial x^2} + k_f v + F(x,t) = 0, \quad 0 \le x \le L_e.$$

with boundary conditions (Clamped–Simply supported):

$$at\ \ x = 0, \quad W(x) = \frac{dW(x)}{dx} = 0$$

$$at\ \ x = L_{e,} \quad W(x) = \frac{d^2W(x)}{dx^2} = 0$$

Solve the Riccati equation problem using the adomian decomposition method (ADM). Then compared the results with exact solutions.

Q3 **(A)** Consider the following Riccati equation [25]

$$y'(t) = -(3 - y(t))^2,$$

with initial conditions:

$$y(0) = 1$$

Solve the Riccati equation problem using the adomian decomposition method (ADM).

(B) Consider the following Initial value problem equation

$$\frac{dy}{dt} = t^3 y^2(t) - 2t^4 y(t) + t^5 + 1$$

with initial conditions:

$$y(0) = 0.$$

Solve the problem using the adomian decomposition method (ADM).

(C) Given the modal of wave equation:

$$\frac{\partial u}{\partial t} = -\alpha\frac{\partial u}{\partial x}$$

Using central differencing and apply the Von Neumann stability analysis to illustrate the application of stability analysis to the three-level FDEs

Q4

(A) State the Classification of Partial Differential Equations? And state the various types of boundary conditions?

(B) Determine the approximate forward difference representation for $\partial^3 f/\partial x^3$ which is of the order (Δx), given evenly spaced grid points f_i, f_{i+1}, f_{i+2} and f_{i+3} by means of:

i) Taylor series expansion.

ii) Forward difference recurrence formula.

iii) A third-degree polynomial passing through the four points.

(C) For the function $f(x) = sin(2\pi x)$, determine $\partial f/\partial x$ at $x = 0.375$ using central difference representation of order $(\Delta x)^2$ and order $(\Delta x)^4$. Use step sizes of 0.01, 0.1 and 0.25. Compare the result with the exact analytical solution and discuss the results.

This exam measures the following ILOs								
Question Number	Q1-a	Q1-b	Q3-b	Q4-a	Q1-c	Q2-a	Q3-a	Q4-c
	Q4-b				Q2-b	Q2-c	Q3-c	
	Knowledge &understanding skills				Intellectual Skills		Professional Skills	

With our best wishes

Dr. Ramzy M. Abumandour